MATH 2171 Reading Day Review - Questions and Source

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1 Source

The following source was used to create the questions for Fall 2022’s Reading Day review session for MATH 2171 (Differential Equations):

*Elementary Differential Equations and Boundary Value Problems 8e* by William E. Boyce and Richard C. DiPrima

Since MATH 2171 final exams are often quite different depending upon one’s professor, the review questions in this document may not cover every topic in one’s specific class, but it should cover the basics that appear in nearly every MATH 2171 class. For each of the given problems, assume that you are solving the differential equation unless specified to do otherwise.

2 *Elementary Differential Equations and Boundary Value Problems 8e* by William E. Boyce and Richard C. DiPrima

2.1 First Order Differential Equations Review Exercises (p. 31-134)

2.1.1 Linear Equations; Method of Integrating Factors

13. \( y' - y = 2te^{2t}, \quad y(0) = 1 \)

14. \( y' + 2y = te^{-2t}, \quad y(1) = 0 \)

15. \( ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, t > 0 \)

16. \( y' + \left( \frac{2}{x} \right)y = \frac{\cos(t)}{t^2}, \quad y(\pi) = 0, t > 0 \)

17. \( y' - 2y = e^{2t}, \quad y(0) = 2 \)

2.1.2 Separable Equations

1. \( y' = \frac{x^2}{y} \)

2. \( y' = \frac{x^2}{y(1+x^2)} \)

3. \( y' + y^2 \sin(x) = 0 \)

4. \( y' = \frac{3x^2 - 1}{3 + 2y} \)

5. \( y' = (\cos^2(x))(\cos^2(2y)) \)
2.1.3 Exact Equations and Integrating Factors

If the given equations are not exact, find an integrating factor to solve them.
1. \((2x + 3) + (2y - 2)y' = 0\)
2. \((2x + 4y) + (2x - 2y)y' = 0\)
3. \((3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0\)
4. \((2xy^2 + 2y) + (2x^2y + 2x)y' = 0\)
8. \((e^x \sin(y) + 3y)dx - (3x - e^x \sin(y))dy = 0\)

2.1.4 Numerical Approximations: Euler’s Method

For the following problems, use \(y' = 3 + t - y, \quad y(0) = 1\). The problems may require different approximations with different step values.
1. Approximate \(y\) at \(t = 0.1\) with \(h = 0.1\).
2. Approximate \(y\) at \(t = 0.2\) with \(h = 0.1\).
3. Approximate \(y\) at \(t = 0.3\) with \(h = 0.1\).
4. Approximate \(y\) at \(t = 0.4\) with \(h = 0.1\).
5. Approximate \(y\) at \(t = 0.1\) with \(h = 0.05\).

2.1.5 Miscellaneous Problems

1. \(\frac{dy}{dx} = \frac{x^3 - 2y}{x}\)
2. \((x + y)dx - (x - y)dy = 0\)
3. \(\frac{dy}{dx} = \frac{2x + y}{3 + x^2 - x}, \quad y(0) = 0\)
4. \((x + e^y)dy - dx = 0\)
5. \(\frac{dy}{dx} = \frac{-2xy + y^2 + 1}{x^2 + 2xy}\)
6. \(x \frac{dy}{dx} + xy = 1 - y, \quad y(1) = 0\)
7. \(\frac{dy}{dx} = \frac{x}{x^2 + y^2}\)
8. \(x \frac{dy}{dx} + 2y = \frac{\sin(x)}{x}, \quad y(2) = 1\)
9. \(\frac{dy}{dx} = \frac{-2xy + 1}{x^2 + 2y}\)
10. \((3y^2 + 2xy)dx - (2xy + x^2)dy = 0\)
2.2 Second Order Linear Equations (p. 135-218)

2.2.1 Homogeneous Equations with Constant Coefficients
1. \( y'' + 2y' - 3y = 0 \)
2. \( y'' + 3y' + 2y = 0 \)
3. \( 6y'' - y' - y = 0 \)
4. \( 2y'' - 3y' + y = 0 \)
5. \( y'' + 5y' = 0 \)

2.2.2 Complex Roots of the Characteristic Equation
7. \( y'' - 2y' + 2y = 0 \)
8. \( y'' - 2y' + 6y = 0 \)
9. \( y'' + 2y' - 8y = 0 \)
10. \( y'' + 2y' + 2y = 0 \)
11. \( y'' + 6y' + 13y = 0 \)

2.2.3 Repeated Roots; Reduction of Order
1. \( y'' - 2y' + y = 0 \)
2. \( 9y'' + 6y' + y = 0 \)
3. \( 4y'' - 4y' - 3y = 0 \)
4. \( 4y'' + 12y' + 9y = 0 \)
5. \( y'' - 2y' + 10y = 0 \)

2.2.4 Nonhomogeneous Equations; Method of Undetermined Coefficients
1. \( y'' - 2y' - 3y = 3e^{2t} \)
2. \( y'' + 2y' + 5y = 3 \sin(2t) \)
3. \( y'' - 2y' - 3y = -3te^{-t} \)
4. \( y'' + 2y' = 3 + 4 \sin(2t) \)
5. \( y'' + 9y = t^2e^{3t} + 6 \)
2.2.5 Variation of Parameters

5. \( y'' + y = \tan(t), \quad 0 < t < \frac{\pi}{2} \)

6. \( y'' + 9y = 9\sec^2(3t), \quad 0 < t < \frac{\pi}{6} \)

7. \( y'' + 4y' + 4y = -t^2e^{-2t}, \quad t > 0 \)

8. \( y'' + 4y = 3\csc(2t), \quad 0 < t < \frac{\pi}{2} \)

9. \( 4y'' + y = 2\sec\left(\frac{t}{2}\right), \quad -\pi < t < \pi \)

2.3 The Laplace Transform (p. 307-354)

2.3.1 Definition of the Laplace Transform

Using the definition of the Laplace Transform, find the Laplace Transform of the given functions.

1. \( t^2 \)

11. \( \sin(bt), \quad b \in \mathbb{R} \)

12. \( \cos(bt), \quad b \in \mathbb{R} \)

15. \( te^{at}, \quad a \in \mathbb{R} \)

16. \( t \sin(at), \quad a \in \mathbb{R} \)

2.3.2 Solution of Initial Value Problems

Use the Laplace transform to solve the given initial value problems.

11. \( y'' - y' - 6y = 0, \quad y(0) = 1, y'(0) = -1 \)

12. \( y'' + 3y' + 2y = 0, \quad y(0) = 1, y'(0) = 0 \)

13. \( y'' - 2y' + 2y = 0, \quad y(0) = 0, y'(0) = 1 \)

14. \( y'' - 4y' + 4y = 0, \quad y(0) = 1, y'(0) = 1 \)

15. \( y'' - 2y' + 4y = 0, \quad y(0) = 2, y'(0) = 0 \)

2.3.3 Step Functions

Find the Laplace transform of the given functions.

7. \( f(t) = \begin{cases} 0, & t < 2 \\ (t - 2)^2, & t \geq 2 \end{cases} \)

8. \( f(t) = \begin{cases} 0, & t < 1 \\ t^2 - 2t + 2, & t \geq 1 \end{cases} \)
9. \[ f(t) = \begin{cases} 
0, & t < \pi \\
 t - \pi, & \pi \leq t < 2\pi \\
0, & t \geq 2\pi 
\end{cases} \]

10. \[ f(t) = u_1(t) + 2u_3(t) - 6u_4(t) \]

11. \[ f(t) = (t - 3)u_2(t) - (t - 2)u_3(t) \]

2.3.4 Differential Equations with Discontinuous Forcing Functions

3. \[ y'' + 4y = \sin(t) - u_{2\pi}(t)\sin(t - 2\pi), \quad y(0) = 0, y'(0) = 0 \]

4. \[ y'' + 4y = \sin(t) + u_{\pi}(t)\sin(t - \pi), \quad y(0) = 0, y'(0) = 0 \]

6. \[ y'' + 3y' + 2y = u_2(t), \quad y(0) = 0, y'(0) = 1 \]

7. \[ y'' + y = u_{3\pi}(t), \quad y(0) = 1, y'(0) = 0 \]

8. \[ y'' + y' + \frac{5}{4}y = t - u_{\frac{\pi}{2}}(t - \frac{\pi}{2}), \quad y(0) = 0, y'(0) = 0 \]