Non-Mandatory Practice for the Final Exam

1. Find the equilibrium point for the supply and demand functions below. Enter your answer as an ordered pair.

\[ S(x) = 3x^2 - 18x \]
\[ D(x) = 90 - 15x \]

Answer: \((x_E, p_E) = \) ______________

2. Find the one-sided limits of \( f(x) \).

\[ f(x) = \begin{cases} 
7x + 7 & \text{if } x < 7 \\
5x^2 - 3 & \text{if } x \geq 7 
\end{cases} \]

**Step 1.** Find \( \lim_{{x \to 7^-}} f(x) \).

Answer: ______________

**Step 2.** Find \( \lim_{{x \to 7^+}} f(x) \).

Answer: ______________

3. Find the limit algebraically by factoring the expression first.

\[ \lim_{{x \to 2}} \frac{5x^2 - 2x - 16}{x - 2} \]

Answer: ______________

4. Suppose \( f(x) \) models rockets produced for input \( x \) in labor units.
Step 1. Interpret the meaning of \( f(900) = 400 \).

A) 400 is the average increase in production caused by increasing labor input from 1 unit to 900 units.

B) An increase in labor input from 900 units to 901 units will cause an increase in production of 400 rockets.

C) 400 is the total change between the first day and 900 days later.

D) An input of 900 labor units has produced 400 rockets.

Step 2. Interpret the meaning of \( f ^ \prime (900) = 16 \).

A) An input of 900 labor units has produced 16 rockets.

B) 16 is the average increase in production caused by increasing labor input from 1 unit to 900 units.

C) An increase in labor input from 900 units to 901 units will cause an increase in production of 16 rockets.

D) 16 is the total change between the first day and 900 days later.

5. Use algebraic techniques to rewrite \( y = \sqrt{x} \left( 8x^\frac{1}{2} + 2x^\frac{3}{2} + 8 \right) \) as a sum or difference; then find \( y ^ \prime \).

Answer: \( y ^ \prime = \) _______________

6. Use algebraic techniques to rewrite \( f (x) = \frac{-3x^8 - 7x^6}{x^5} \) as a sum or difference; then find \( f ^ \prime (x) \).

Answer: \( f ^ \prime (x) = \) _______________
7. A particle is traveling. It is \( g(x) = 0.4x^2 - 3x \) feet off the ground after \( x \) seconds.

**Step 1.** Find the rate of change of the particle's position at \( x = 4 \) seconds.

Answer: _______________

**Step 2.** When will the rate of change be 1?

Answer: _______________

8. A sales representative for a company that produces electric can openers can sell \( x \) units of their deluxe model if the price is \( p = D(x) = 84.6 - 0.01x \) dollars. The total cost for these electric can openers is given by \( C(x) = 0.05x^2 + 5.2x + 5700 \) dollars. Determine the marginal profit for 124 electric can openers.

Answer: \( P'(124) = \) _______________

9. Find the limit.

\[
\lim_{{x \to -\infty}} \frac{4x^2 + 5}{-3x^2 - 6}
\]

Answer: _______________

10. Use the graph of \( y = f(x) \) to answer the question regarding the function.
Step 1. Find \( \lim_{x \to 3^-} f(x) \).
A) ___________  B) Does Not Exist

Step 2. Find \( \lim_{x \to 3^+} f(x) \).
A) ___________  B) Does Not Exist

Step 3. Find \( f(3) \).
A) ___________  B) Does Not Exist

Step 4. Is \( f(x) \) continuous at \( x = 3 \)?
A) Yes
B) No
11. Given \( f(4) = 9, f'(4) = -8, g(4) = 2, \) and \( g'(4) = -17, \) find the value of \( h'(4) \) based on the function below.

\[
h(x) = \frac{f(x)}{g(x)}
\]

Answer: \( h'(4) = \quad \)

12. Given \( f(x) = (2x + 3)(-2x + 4), \) find the \((x, y)\)-coordinate on the graph where the tangent line is horizontal.

Answer: \( (\quad, \quad) \)

13. Find the derivative for the given function. Write your answer using positive and negative exponents instead of fractions and use fractional exponents instead of radicals.

\[
f(x) = (10x^4 - 9)^3
\]

Answer: \( f'(x) = \quad \)

14. Find the derivative for the given function. Write your answer using positive and negative exponents and fractional exponents instead of radicals.

\[
g(t) = \frac{7}{\sqrt{t^2 + 5}}
\]

Answer: \( g'(t) = \quad \)

15. It is estimated that \( t \) years from now the population of a city will be \( P(t) = 30(35 + 3t)^2 - 1700t. \)

**Step 1.** What will the population be in 6 years?
Answer: _______________ people

**Step 2.** Find the rate of change in population in 8 years.

Answer: _______________ people/year

16. Consider the function.

\[ f(x) = 4x^3 - 24x^2 - 60x \]

**Step 1.** Find all values of \( x \) that correspond to horizontal tangent lines. Select "None" if the function does not have any values of \( x \) that correspond to horizontal tangent lines.

A) \( x = \) ____________  

B) None

**Step 2.** Determine the open intervals on which the function is increasing and on which the function is decreasing. Enter \( \emptyset \) to indicate the interval is empty.

Answer: 

Increasing: _______________

Decreasing: _______________

17. Consider the function:

\[ f(x) = 4x^3 + 39x^2 - 42x - 10 \]

**Step 1.** Find the critical values of the function. Separate multiple answers with commas.

A) \( x = \) ____________  

B) None

**Step 2.** Use the First Derivative Test to find any local extrema. Enter any local extrema as an ordered pair.

A) Local Maxima: ____________  

B) No Local Maxima

A) Local Minima: ____________  

B) No Local Minima
18. Consider the function \( f(x) = -9x^2 + 54x + 1 \) on the interval \([-4, 6]\). Find the absolute extrema for the function on the given interval. Express your answer as an ordered pair \((x, f(x))\).

Answer: Absolute Maximum: _______________
Absolute Minimum: _______________

19. Consider the function:
\[ f(x) = -8x^3 - 72x^2 + 7x - 5 \]

**Step 1.** Determine the intervals on which the function is concave upwards or concave downwards.

A) Concave Up: ____________  B) Never Concave Up

A) Concave Down: ____________  B) Never Concave Down

**Step 2.** Locate any points of inflection. Enter your answer as \((x, y)\)-pairs.

A) Points of Inflection: ____________  B) None

20. Donnie's father is planning to open a savings account to pay for Donnie's college education. He has found a bank that will pay 9 percent interest compounded annually. How much will he need to deposit initially so that in 16 years the balance will be $46,000? Round your answer to the nearest cent.

Answer: $_______________

21. Use the properties of logarithms to expand the logarithmic expression as much as possible into the sum and/or difference of logarithmic expressions. **Note:** No term should contain exponents.
\[ \ln\left(\frac{x}{\sqrt{x^2+y^2}}\right) \]

Answer: _________________

22. Use the properties of logarithms to write the logarithmic expression as a single logarithm with no coefficients. Simplify.

\[ 3 \ln(5) + \ln(x) + \ln(23) \]

Answer: _________________

23. Solve the following equation for \( x \):

\[ \ln\left((x-4)(x-5)\right) - \ln(x-4) = \ln(9) \]

Answer: \( x = \) _________________

24. How long does it take for $4050 to double if it is invested at 9% compounded continuously?

Round your answer to two decimal places.

Answer: _________________ years

25. Solve the following logarithmic equation using a calculator, if necessary, to evaluate the logarithm. Enter your answer as a fraction or round your answer to two decimal places. \( e = 2.7183 \)

\[ \ln(7x + 12) = 4 \]

Answer: \( x = \) _________________
26. Solve the following exponential equation for $x$. Write the answer as both an exact expression and as a decimal approximation (rounded to 2 decimal places).

$$2e^{5x + 3} = 11$$

$$\approx \underline{\text{}} \quad x = \underline{\text{}}$$

27. A carpenter has determined that the cost of producing $x$ chairs is given by the function $C(x) = 69x + \ln(3x^4 + 14)$ dollars. Find the marginal cost function $C'(x)$.

Answer: $C'(x) = \underline{\text{}}$

28. Find the derivative of the given expression.

$$f(x) = 11x^4 \ln(x^4)$$

Answer: $f'(x) = \underline{\text{}}$

29. Find the derivative of the given expression.

$$f(x) = \frac{\ln(x^4)}{x^4}$$

Answer: $f'(x) = \underline{\text{}}$

30. Find the derivative of the given expression.

$$f(x) = 11(\ln(x))^5$$

Answer: $f'(x) = \underline{\text{}}$
31. Find the following indefinite integral.

\[ \int \left( \frac{5}{x} + 7x^{-5} + 5e^x \right) dx \]

Answer: 

32. Use integration by substitution to solve the integral below. Use \( C \) for the constant of integration.

\[ \int (2x^2 - 5x + 1)^2(4x - 5) dx \]

Answer: 

33. Evaluate the definite integral below.

\[ \int_{8}^{6} dx \]

Enter your answer in exact form or rounded to two decimal places.

Answer: 

34. Evaluate the definite integral below.

\[ \int_{-5}^{-3} (2x + 3) dx \]

Enter your answer in exact form or rounded to two decimal places.

Answer: 

35. Find a formula for \( y' \) and determine the slope \( y' \bigg|_{x=1} \) for the following function.
\[ y = -6e^{2x^3 + 8} \]

Answer: \[ y' |_{x=1} = \] 

36. The marginal profit for a certain style of sunglasses is given by \( P'(x) = 44 - 0.31x \) dollars per pair, where \( x \) is the number of pairs of sunglasses produced and sold weekly. Find the profit for the first 36 pairs of sunglasses that are produced and sold. Round your answer to the nearest cent.

Answer: \( \$ \) 

37. Find the total area bounded by the \( x \)-axis and the curve \( y = f(x) \) on the indicated interval.

\[ f(x) = x^2 - x - 12; [3,6] \]

Answer: \( \)